Otimização em Engenharia/Optimization in Engineering – 15235

2024/2025

Mini-project 3
Minimization of the mass of a beam

1. OBJECTIVE

To learn how to implement a computer code using gradient-free optimization algorithms to solve a constrained minimization problem.

2. PROBLEM

A built-in beam with a length (L) and a rectangular cross-section of width (w) and height (h) is loaded with a vertical tip force (P). The properties of the material used in its construction are a Young's modulus (E) and a density (ρ).

Write a computer code that determines the cross-section size which minimizes the mass of the beam when it is subjected to the tip force, complying with a maximum direct stress (σ_{max}), a maximum shear stress (τ_{max}), a maximum tip deflection (δ_{max}), a minimum width (w_{min}), a minimum height (h_{min}), a maximum width (w_{max}), and a maximum height (h_{max}). The equations for this problem are given below.

$$m = \rho L w h \quad ; \quad \sigma_{max} = \frac{6PL}{wh^2} \quad ; \quad \tau_{max} = \frac{3P}{2wh} \quad ; \quad \delta_{max} = \frac{4PL^3}{Ewh^3}$$

Plot the mass as a function of the cross-section dimensions, b and h, and the obtained solution.

The following tasks are required to develop the proposed work:

a) Select an optimization algorithm from Table 2, implement it in a computer code using a programming language of your choice and validate it using the 2-variable Rosenbrock function: $f(x, y) = (a - x)^2 + b(y - x^2)^2$ with $a = 1$ and $b = 100$ and $x \in [-2,2]$ and $y \in [-1,3]$.

b) Define and implement a computer code to calculate the optimization function mass (m) and the constraints direct tress (σ), shear stress (τ) and tip deflection (δ), given the design variables width (w) and height (h).

c) Define and implement the optimization problem of the beam.

d) Minimize the mass of the beam using the numerical tools previously developed.
e) Write a report containing the following: problem description, solution methodology, code implementation, results and discussion, conclusions. The developed code should be placed in an appendix.

3. REPORT

Each team of 3 to 4 students (team from Table 3, using different data from Tables 1 and 2) must present the results of their work in a written report, **with a maximum of 20 pages**. It should explain in adequate detail all the steps associated with solving the problem. A critical analysis of the results should also be presented.

The report should follow the following structure:
a) Introduction: presentation of the problem and objectives. [10 points]
b) Optimization algorithm: brief description, implementation, and validation. [60 points]
c) Beam analysis: objective and constraint functions, implementation. [40 points]
d) Beam optimization: minimization of the beam's mass. [40 points]
e) Analysis and Discussion: evaluation of the results and critical analysis. [40 points]
f) Conclusions: considering the objectives, overall evaluation of the work and the obtained results. [10 points]

The deadline for submitting the report is December 20th, 2024. The report in *pdf* format must be sent via email to pgamboa@ubi.pt.

4. DATA

Each team of 3 to 4 students will select a different case from Table 1.

Table 1. Beam loading and characteristics.

Case	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
L, m	0.9	0.9	0.9	1	1	1	1.1	1.1	1.1	1.2	1.2	1.2
E, GPa	70	140	210	70	140	210	70	140	210	70	140	210
ρ, kg.m^{-3}	2700	1500	7900	2700	1500	7900	2700	1500	7900	2700	1500	7900
P, kN	3	3	3	3	3	3	3	3	3	3	3	3
σ_{max}, MPa	100	300	200	100	300	200	100	300	200	100	300	200
τ_{max}, MPa	10	5	20	10	5	20	10	5	20	10	5	20
δ_{max}, mm	10	10	10	10	10	10	10	10	10	10	10	10
w_{min}, mm	10	10	10	10	10	10	10	10	10	10	10	10
w_{max}, mm	100	100	100	100	100	100	100	100	100	100	100	100
h_{min}, mm	20	20	20	20	20	20	20	20	20	20	20	20
h_{max}, mm	150	200	250	150	200	250	150	200	250	150	200	250

Table 2 – Optimization algorithm.

#	Algorithm
1	Nelder–Mead
2	generalized pattern search (GPS)
3	DIRECT
4	genetic algorithm (GA) bit-encoded
5	genetic algorithm (GA) real-encoded
6	particle swarm optimization (PSO)

Table 3 – Teams.

Team	Case	Alg.	Student 1	Student 2	Student 3	Student 4
1	2	4	Pedro Leite M14197	Anton Mamus M14212	Pedro Moreira M14298	Francisco Ribeiro E11572
2	6	6	Miguel Ruivo M14234	Alexandre Teixeira M14291	Isabel Gomes M14292	Miguel Albino M14293
3	5	5	Afonso Gamboa M14208	Daniel Câmara M14445	Sebastião Ventura M14473	Breno Cabral
4	4	3	André Sousa E11575	Maria Fernandes M14297	Octávio Lopes M14517	Cristina Martins M15038
5	3	2	Diogo Pinho M14214	Beatriz Gonçalves M14442	Pedro Rafeiro M14527	Riaan Rasga M14953
6	1	1	João Ramalhosa M14135	Mariana Peixoto M14179	Gonçalo Dias M14443	Júlia Russo A55210