UNIVERSIDADE DA BEIRA INTERIOR
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Introduction to Aeroelasticity

Estruturas Aeroespaciais II (10373/15095)

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1. Introduction

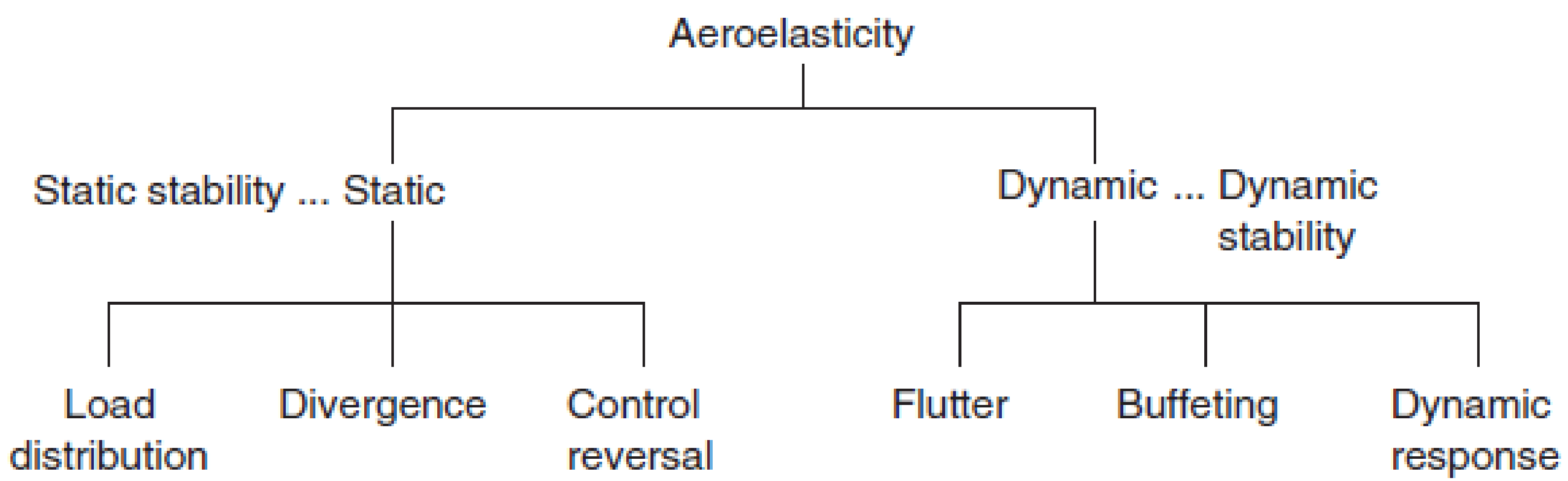
There are several phenomena that affect the aero-structural behaviour of lifting surfaces.

Therefore, the design of the lifting surfaces must ensure that those effects do not occur within the normal flight envelope of an aircraft.

The three phenomena which affect the lifting surfaces of aircraft are:

1. Flutter
2. Divergence
3. Control reversal

1. Introduction

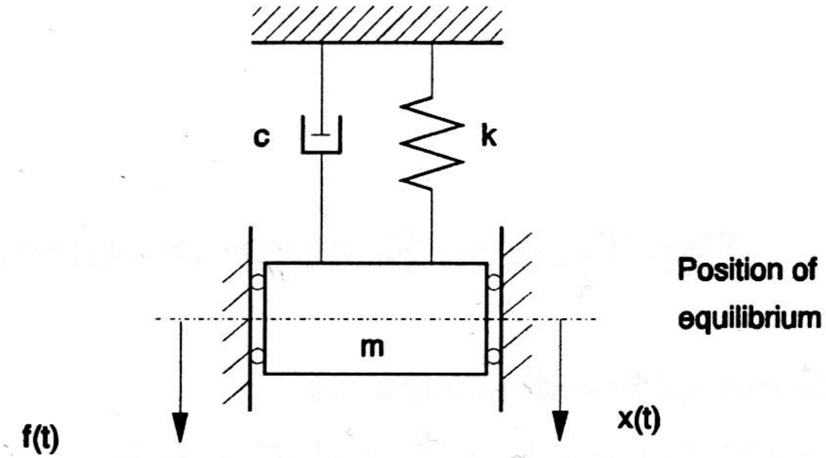


1. Introduction
Single-Degree-of-Freedom Stability

Let us consider a mechanical system with only one degree-of-freedom.

This system consists of a concentrated mass m, a spring with constant k and a dash-pot with a viscous damping coefficient c.

The external force applied is $F(t)$ and the displacement $x(t)$ is measured from the equilibrium position.



1. Introduction
Single-Degree-of-Freedom Stability

The equation of motion of the system is

$$mx'' + cx' + kx = F(t)$$

where ´ represents time derivatives.

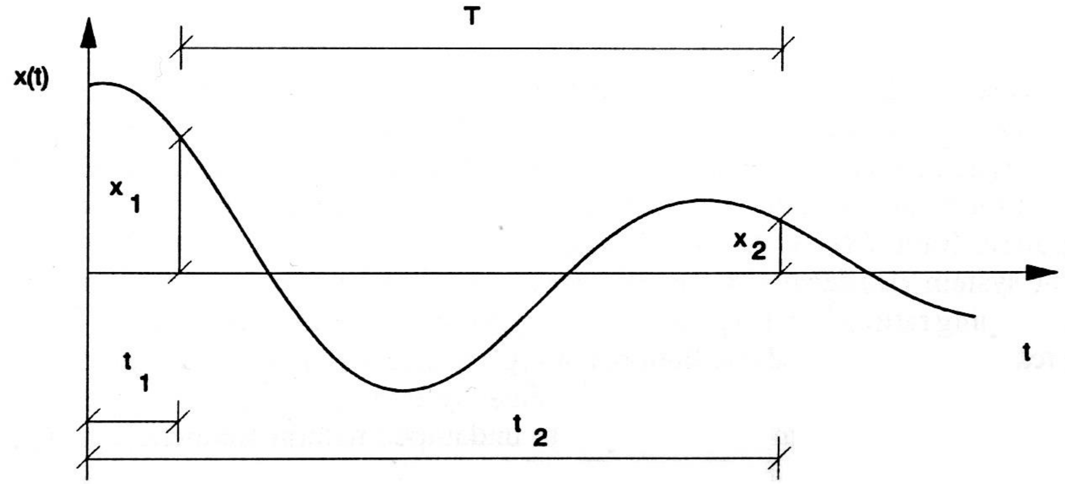
In a dynamic structural system, *m*, *c* and *k* are real positive constants.

This system is stable, in the sense that, if it is subjected to an initial perturbation after, for example, an impulsive force $F(t)=F_0\delta(t)$ where $\delta(t)$ is the Dirac delta function is applied to it, the response of the system $x(t)$ reduces asymptotically to zero.

1. Introduction
Single-Degree-of-Freedom Stability

Let us define the damping ratio as $\xi = c/c_{cr} = c/(4km)^{1/2}$, where c_{cr} is the critical damping coefficient.

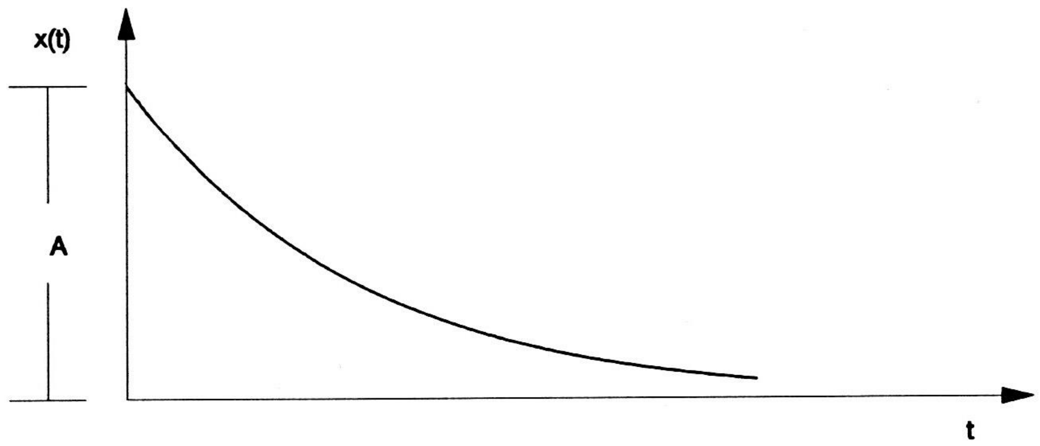
Thus, when $\xi^2 < 1$ or $c^2 < 4km$, the response is a damped oscillation with mechanical system damped frequency $\omega_d = \omega_0(1-\xi^2)^{1/2}$, where $\omega_0 = (k/m)^{1/2}$ is the non-damped natural frequency of the mechanical system.



1. Introduction
Single-Degree-of-Freedom Stability

When $c^2>4km$, the response is also stable, but the motion is non-oscillatory.

The figure below shows a typical response in this case.



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1. Introduction
Single-Degree-of-Freedom Stability

Let us now consider the same mechanical system but with some extra forces added: non-conservative forces (forces which do not depend on a potential, and are inherent to the system, i.e., they are not loads externally applied due to the change in position of the generalized coordinate).

In aeroelasticity, these forces are incremental aerodynamic loads due to the motion.

Assuming that these forces are proportional to the displacement and to the speed of the system, the equation of motion becomes

$$mx'' + cx' + kx = F(t) + c_1 x' + k_1 x$$

1. Introduction
Single-Degree-of-Freedom Stability

or

$$m x'' + c_{eff} x' + k_{eff} x = F(t)$$

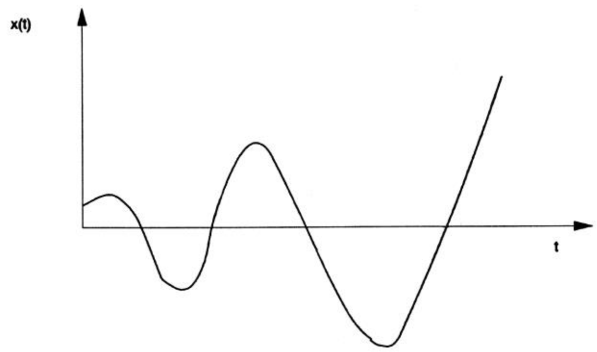
where $c_{eff}=c-c_1$ is the effective damping of the system configuration and $k_{eff}=k-k_1$ is the effective stiffness of the system configuration.

For $k_1=0$ and $c_1 \neq 0$:

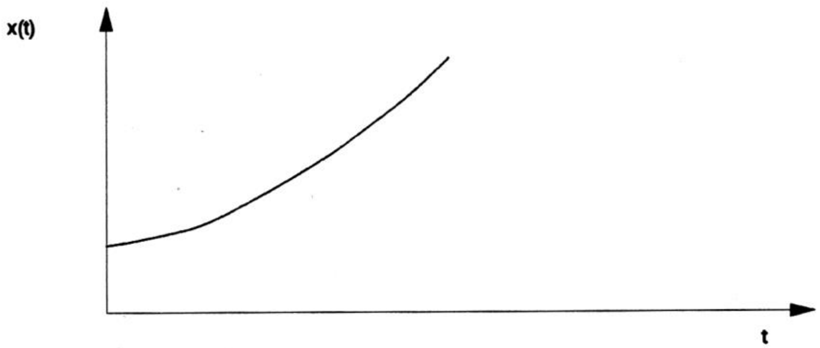
- if c_1 is real positive and less than c, the behaviour of the system is as before and stable.
- if c_1 is real positive and greater than c, then damping is negative and the response is divergent (unstable).

1. Introduction
Single-Degree-of-Freedom Stability

For $c_{eff}^2 < 4k_{eff}m$ the motion is oscillatory and divergent:



For $c_{eff}^2 > 4k_{eff}m$ the motion is non-oscillatory and divergent:



2. Flutter

Flutter is a dangerous phenomenon encountered in flexible structures subjected to aerodynamic forces.

These include: aircraft, buildings, telegraph wires, traffic signs, and bridges.

Flutter occurs as a result of interactions between aerodynamic, elastic, and inertial forces on a structure.

In an aircraft, as the speed of the wind increases, there may be a point at which the structural damping is insufficient to damp out the motions which are increasing due to aerodynamic energy being added to the structure.

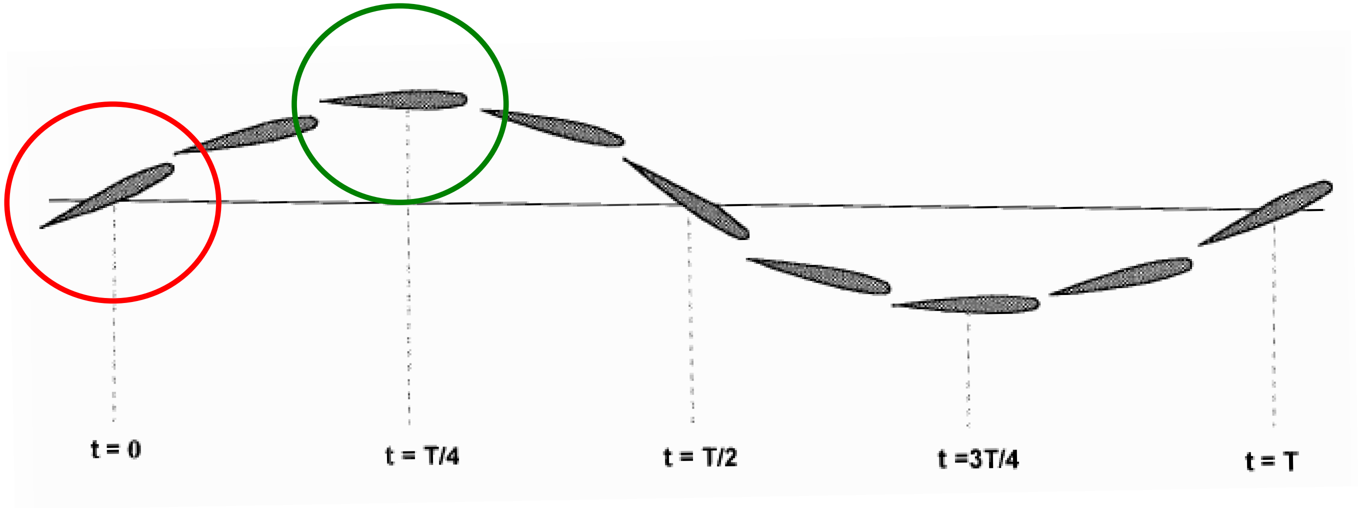
This vibration can cause structural failure and therefore considering flutter characteristics is an essential part of designing an aircraft.

2. Flutter
Flutter Motion

The basic type of flutter of an aircraft wing is described here.

Flutter may be initiated by a rotation of the airfoil (t=0).

As the increased force causes the airfoil to rise, the torsional stiffness of the structure returns the airfoil to zero rotation (t=T/4).

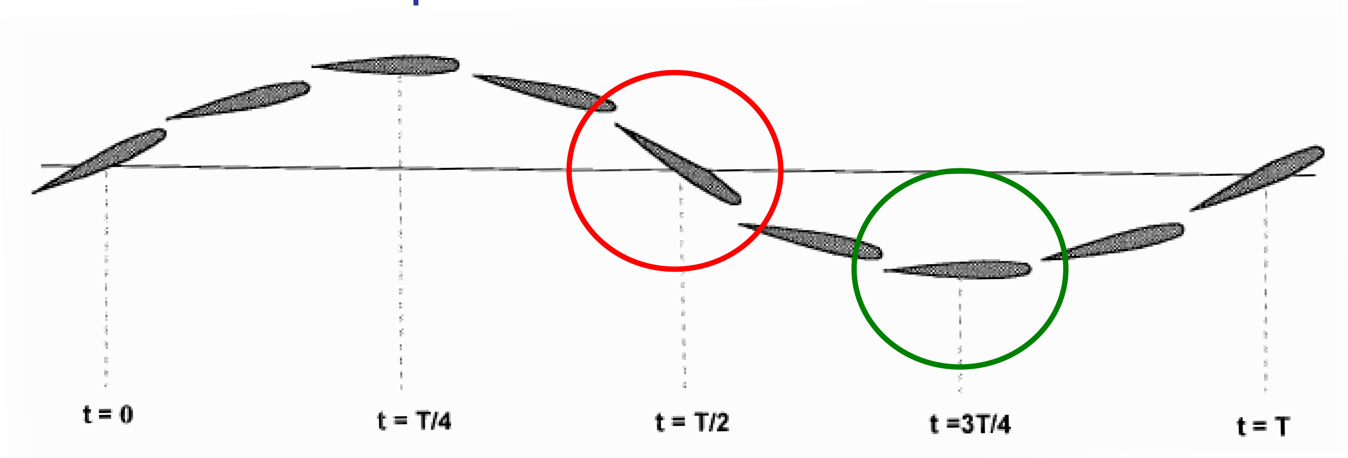


2. Flutter
Flutter Motion

The bending stiffness of the structure tries to return the airfoil to the neutral position, but now the airfoil rotates in a nose-down fashion ($t=T/2$).

Again, the increased down force causes the airfoil to plunge and the torsional stiffness returns the airfoil to zero rotation ($t=3T/4$).

The cycle is completed when the airfoil returns to the neutral position with a nose-up rotation.

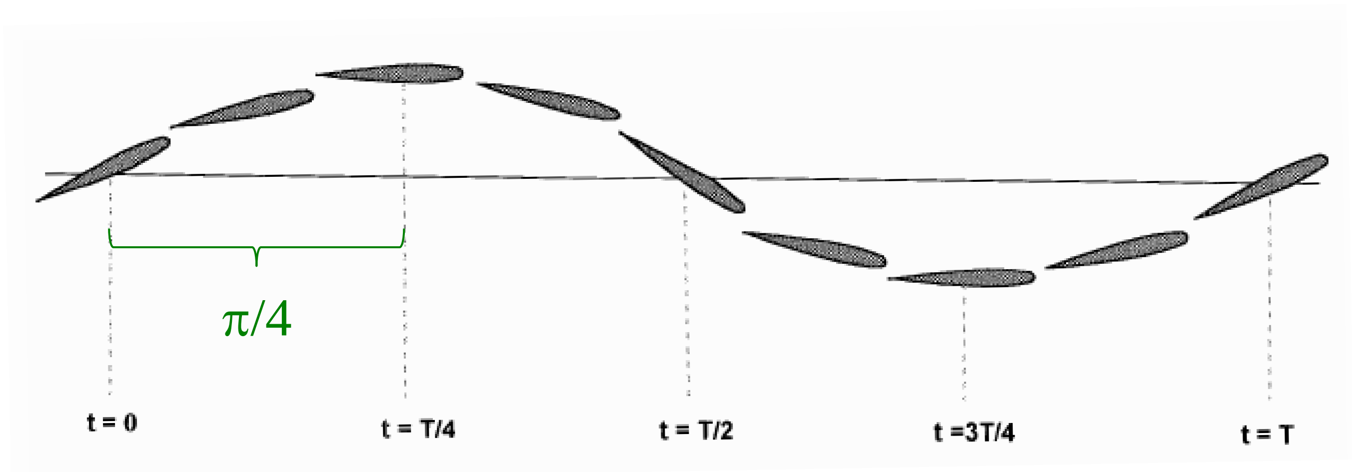


2. Flutter
Flutter Motion

Notice that the maximum rotation leads the maximum rise or plunge by 90 degrees ($T/4$).

As time increases, the plunge motion tends to damp out, but the rotation motion diverges.

If the motion is allowed to continue, the forces due to the rotation will cause the structure to fail.



vídeo

2. Flutter
Flutter Motion

This flutter is caused by the coalescence of two structural modes – pitch and plunge (or wing-bending) motions.

This example wing has two basic degrees of freedom or natural modes of vibration: pitch and plunge (bending).

The pitch mode is rotational and the bending mode is a vertical up and down motion at the wing tip.

As the airfoil flies at increasing speed, the frequencies of these modes coalesce or come together to create one mode at the flutter frequency and flutter condition.

This is the flutter resonance.

2. Flutter
Types of Flutter

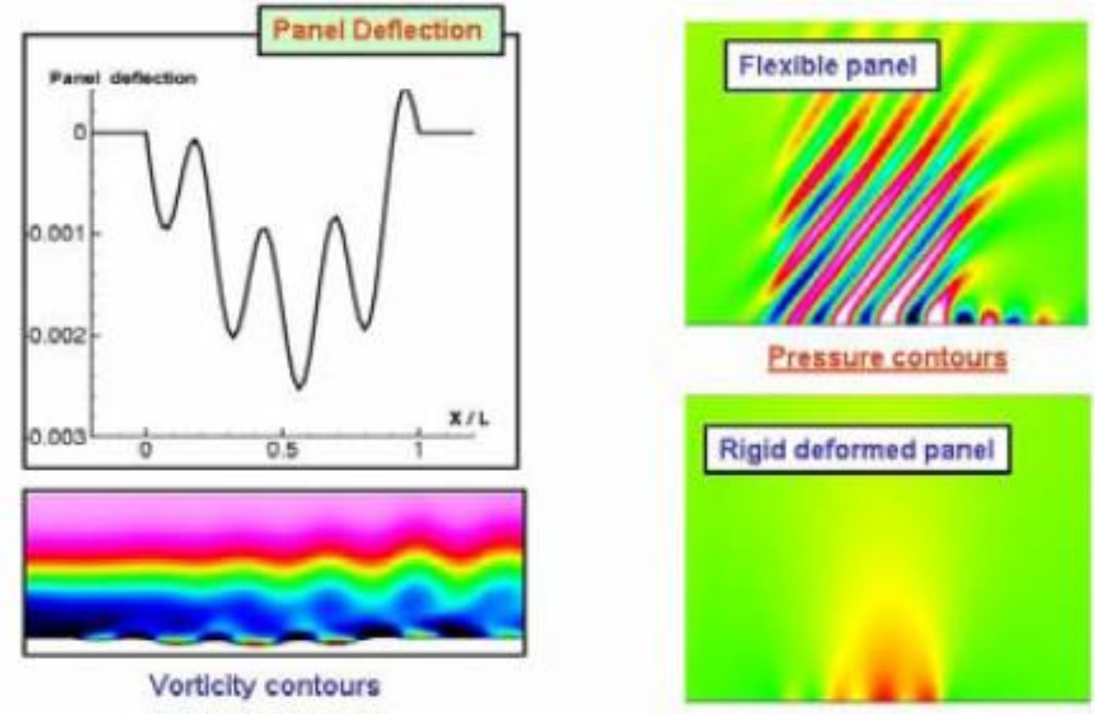
There are many types of flutter behaviour that must be considered when designing aircraft:

- panel flutter
- galloping flutter
- stall flutter
- limit cycle oscillations (LCO) or buzz
- propeller or engine whirl flutter
- classical flutter

There can also be flutter due to stores mounted on the wing.

2. Flutter
Types of Flutter

Panel flutter can occur when a surface is not adequately supported (think of the skin of an airplane acting like a drumhead).



2. Flutter
Types of Flutter

Galloping flutter, or wake vortex flutter, was the cause of failure of the Tacoma Narrows Bridge.

This phenomenon can be observed frequently along the roadside when telephone and power lines “gallop” due to strong winds.

You may also observe car radio antenna aerials whipping under certain driving speeds.

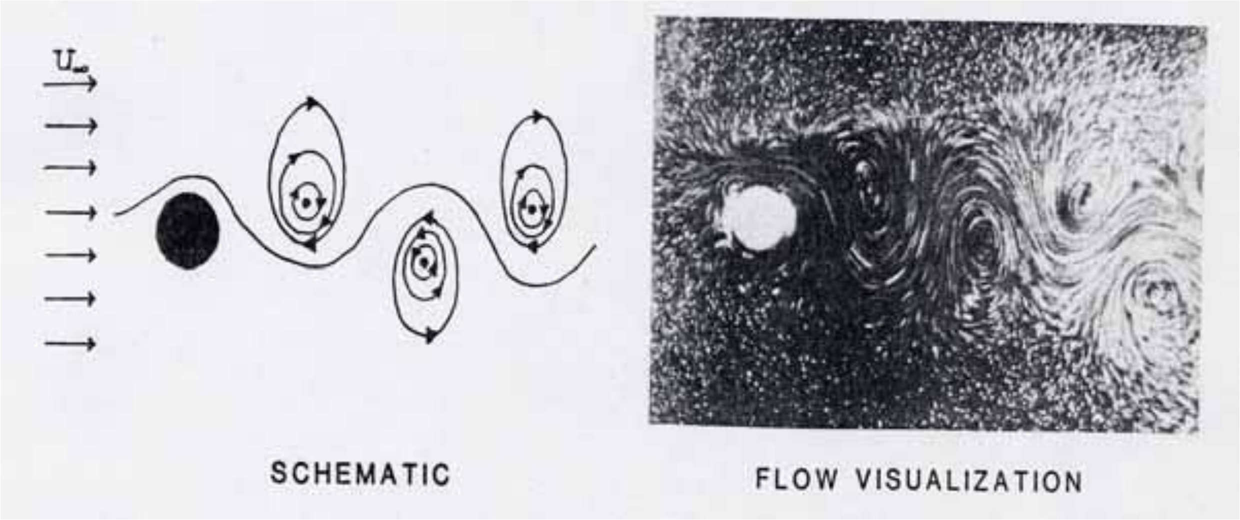
This type of flutter is an important design consideration for launch vehicles exposed to ground winds.

2. Flutter
Types of Flutter

The cause of the galloping motion is formation of wake vortices downstream of the object.

The vortices are shed alternately from one side of the object and then the other.

These cause oscillatory forces and produce the back-and-forth motion.



2. Flutter
Types of Flutter

Stall flutter is a torsion mode of flutter that occurs on wings at high loading conditions near the stall speed.

Because the airflow separates during stall, this single degree-of-freedom flutter cannot be explained by classical flutter theory.

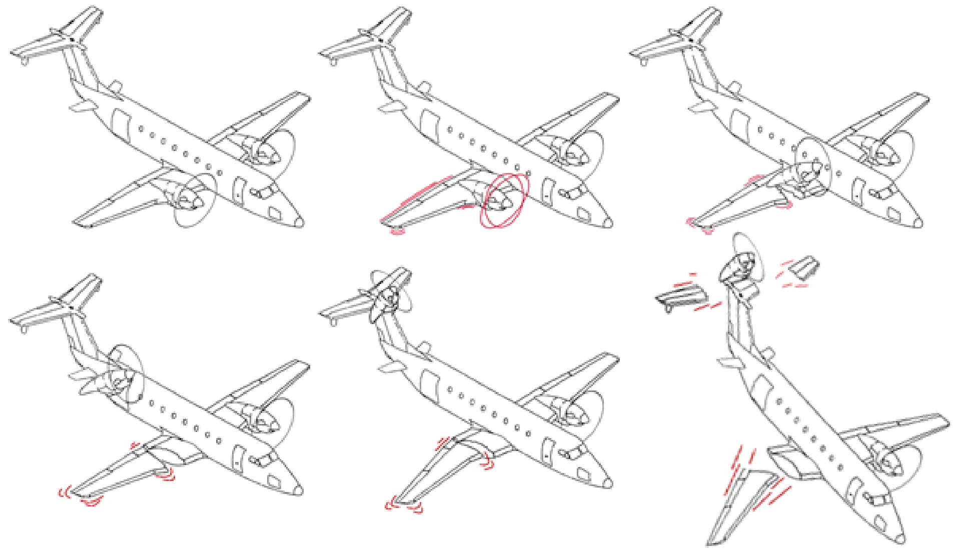
Limit cycle oscillation (LCO) behaviour is characterized by constant amplitude, periodic structural response at frequencies that are those of the aeroelastically-loaded structure.

LCO is typically limited to a narrow region in Mach number or angle-of-attack signalling the onset of flow separation.

2. Flutter
Types of Flutter

Engine whirl flutter is a precession-type instability that can occur on a flexibly mounted engine-propeller combination.

The phenomenon involves a complex interaction of engine mount stiffness, gyroscopic torques of the engine and propeller combination, and the natural flutter frequency of the wing structure.



vídeo

2. Flutter
Classical Flutter

As early aircraft were able to fly at greater speed, flutter may have caused many crashes.

The flutter phenomenon was first identified in 1918 on a Handley Page O/400 bomber in Lanchester, England.

The flutter mechanism consisted of a coupling of the fuselage torsion mode with an anti-symmetric elevator rotation mode.

The elevators on this airplane were independently actuated.

The solution to the problem was to interconnect the elevators with a torque tube.

2. Flutter
Classical Flutter

Scientists and engineers studied flutter and developed theories for the cause and mathematical tools to analyze the behaviour.

In the 1920s and 1930s, unsteady aerodynamic theory was developed.

Closed-form solutions to simple, academic problems were studied in the 1940s and 1950s.

In the next thirty years, strip theory aerodynamics, beam structural models, unsteady lifting surface methods (e.g. double-lattice) and finite element models expanded analysis capabilities.

The advent of digital computers has further supported the development of other powerful methods.

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2. Flutter
Classical Flutter

Disciplines involved in analyzing flutter include aerodynamics, structural finite element modeling, control theory (specifically aeroservoelasticity), and structural dynamics.

The following example of a simple two degree-of-freedom model is fundamental to understanding flutter behavior.

Aerodynamic forces excite the structural spring/mass system.

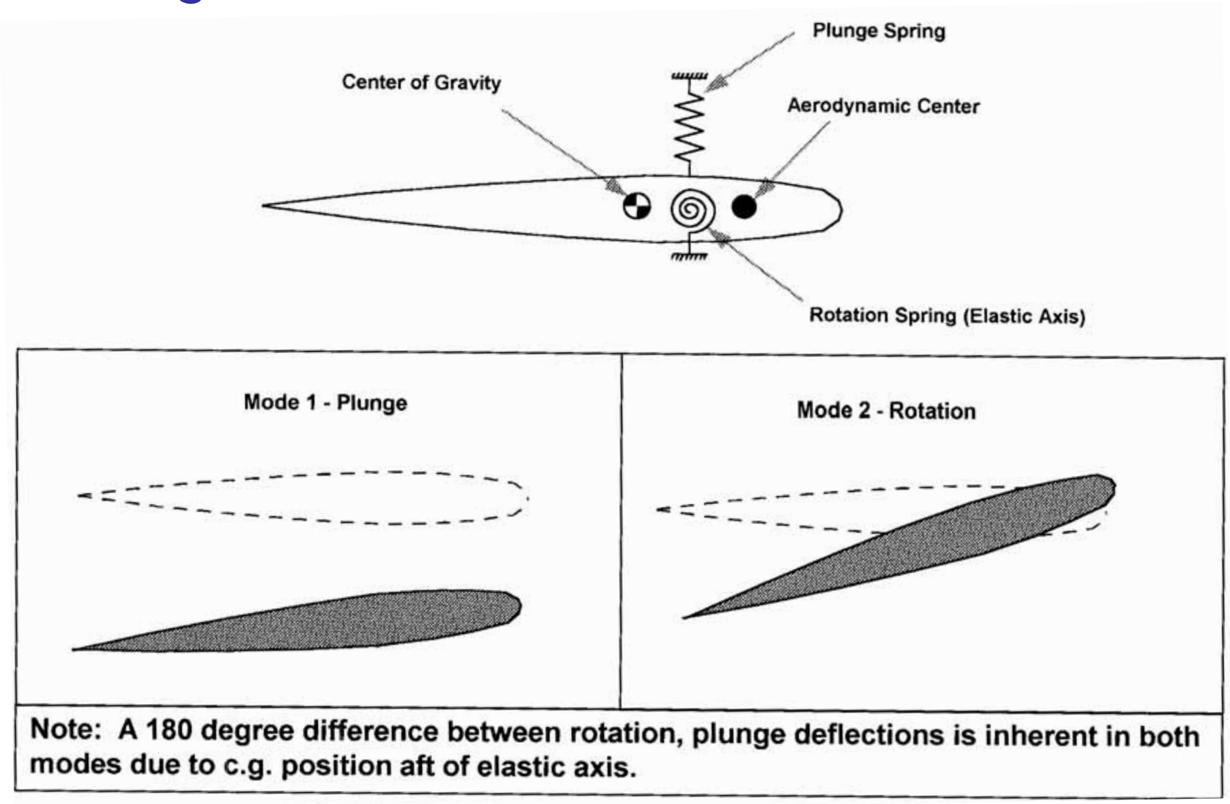
The plunge spring represents the bending stiffness of the structure and the rotation spring represents the torsional stiffness.

The shape of the airfoil determines the aerodynamic center.

The center of gravity is determined by the mass distribution of the cross-section (that is, how the airfoil is constructed).

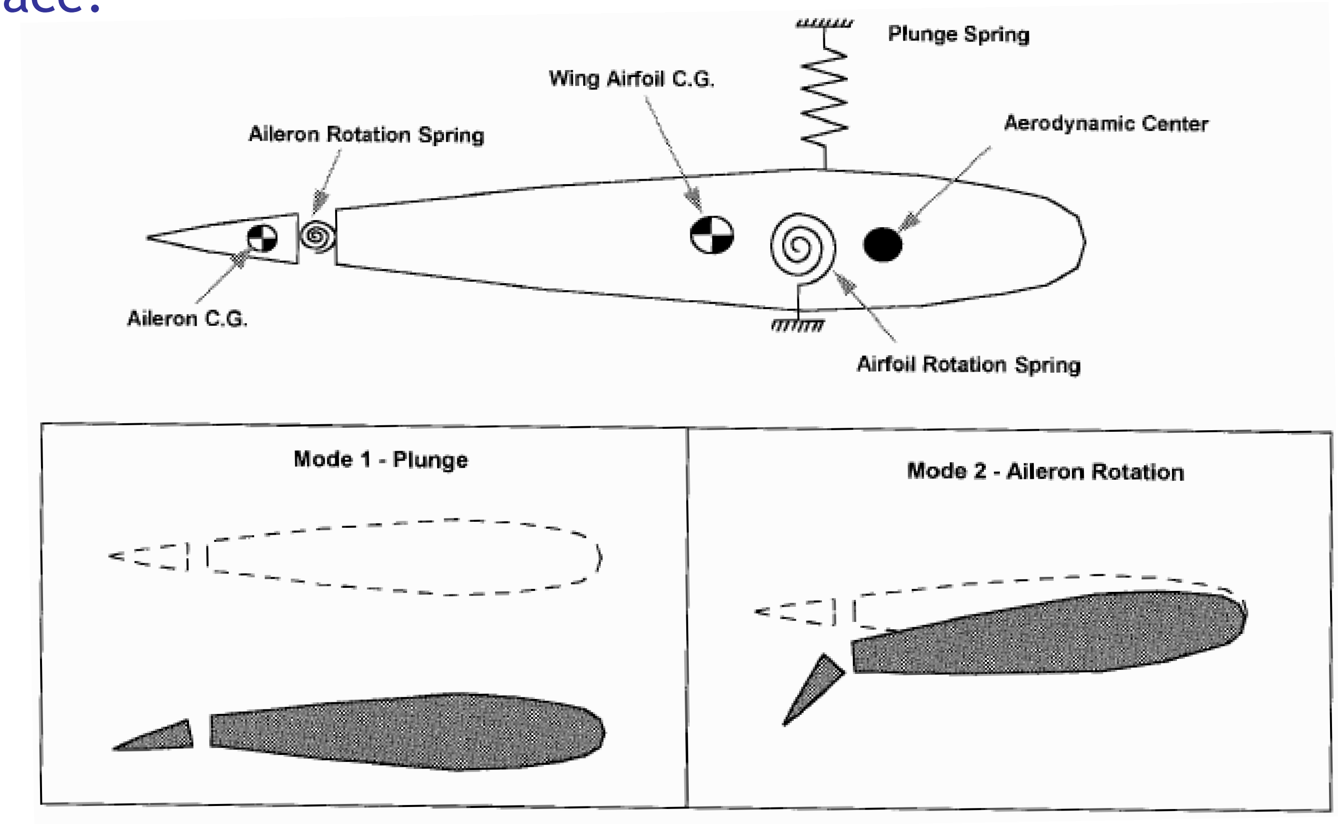
2. Flutter
Classical Flutter

The model represents two “modes” – plunge and rotation as shown in the figure.



2. Flutter
Classical Flutter

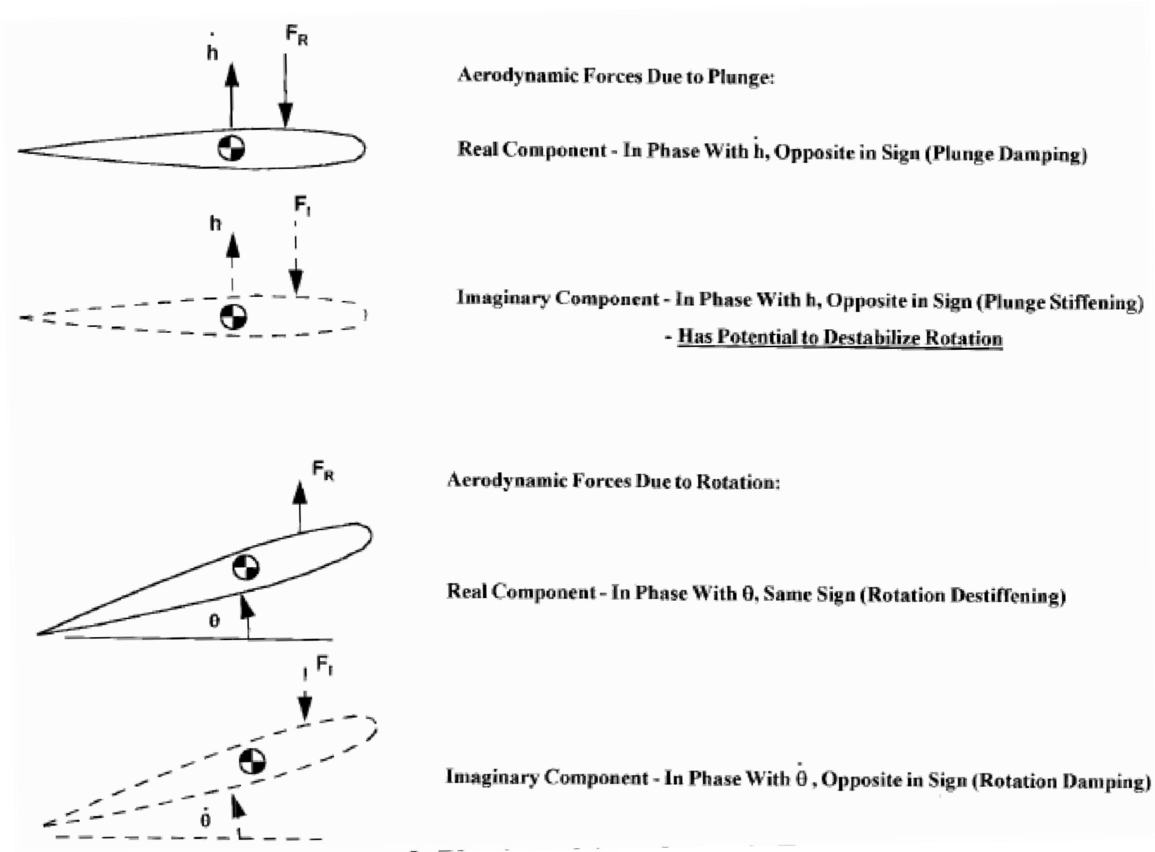
The figure shows a similar model for an airfoil with a control surface.



video

2. Flutter
Classical Flutter

The aerodynamic forces are illustrated in the figures below.



2. Flutter
Flutter Equation of Motion

In the flutter motion of a wing without control surface (or very stiff) one can assume two DOF: plunge (bending) and rotation (torsion).

Therefore, the system's motion is described by a system of two coupled equations

$$mh'' + S_w\theta'' + k_h h + c_h h' = L_{ea}$$
$$I_\theta\theta'' + S_w h'' + k_\theta\theta + c_\theta\theta' = M_{ea}$$

Since at the critical flutter condition damping is zero, then the motion is harmonic and the solution is of the form

$$h = \underline{h}_0 e^{i\omega t} \qquad \theta = \underline{\theta}_0 e^{i\omega t}$$

2. Flutter
Flutter Equation of Motion

Which leads to a system of equations in the form

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} \underline{h_0} \\ \underline{\theta_0} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The solution arises by solving the determinant for the non-trivial solution

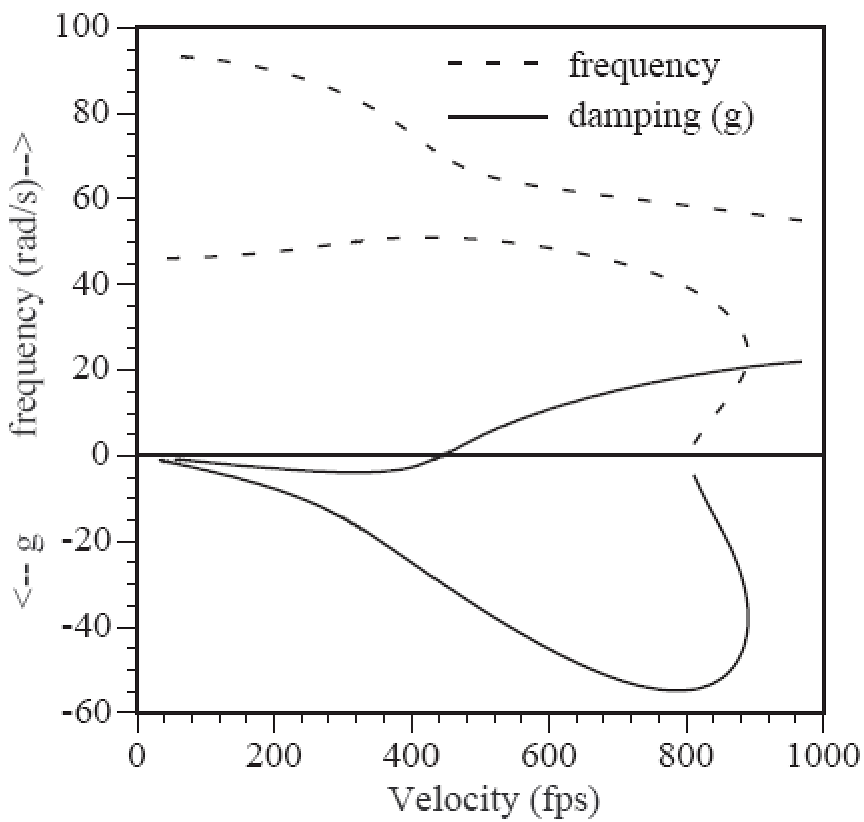
$$\Delta = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = \Delta_R + i\Delta_I = 0$$

2. Flutter
Flutter Equation of Motion

One common form of flutter analysis is the V-g analysis.

In V-g analysis, the structural damping of all the modes of vibration is assumed to have one unknown value, g.

The results for two modes (roots of the flutter determinant) of the simple wing model with 2 degrees of freedom are shown in the form of frequency versus velocity and damping versus velocity curves.



2. Flutter

How to Prevent Flutter?

Because flutter can be analyzed, designs can be modified to prevent flutter before an aircraft is built, tested and flown.

One design parameter is the maximum air speed.

In particular, the ratio of the energy input to the energy dissipated will depend on the air speed.

A steady oscillation may occur when this ratio is unity.

The air speed for this case is called the “critical flutter air speed”.

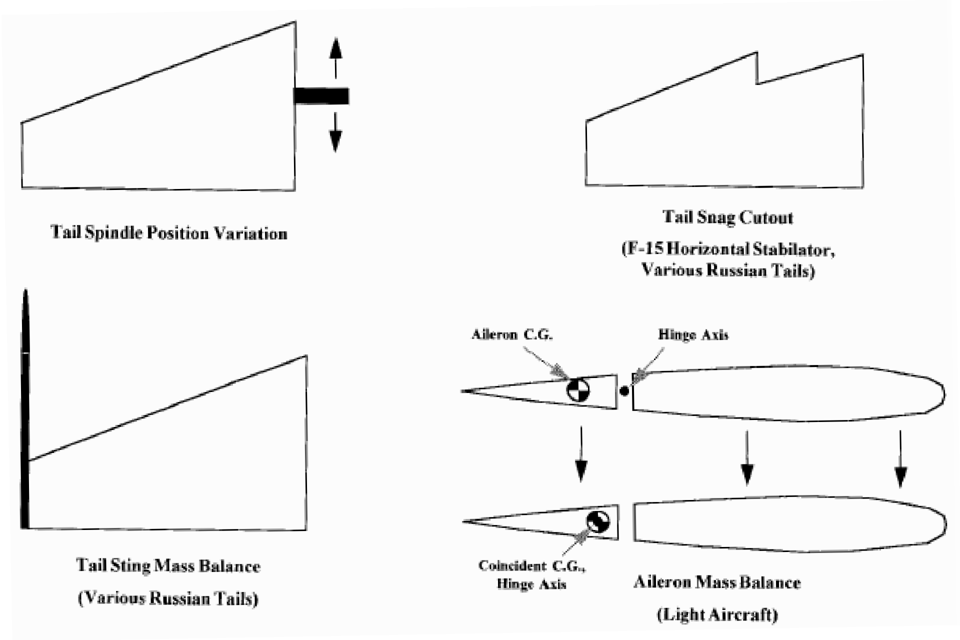
An aircraft may have various possible flutter modes.

Ideally, the lowest critical speed exceeds the highest possible flying speed by a reasonable safety margin.

2. Flutter
How to Prevent Flutter?

There are several additional measures to prevent flutter.

One method is to uncouple the torsion and bending motion by modifying the mass distribution to move the center of gravity closer to the center of twist.



2. Flutter

How to Prevent Flutter?

Another method is to increase the stiffness/mass ratios within the structure.

This would increase the natural frequencies.

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Note that the energy input per cycle during flutter is nearly independent of frequency.

The energy dissipated per cycle is proportional to frequency, however.

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2. Flutter
How to Prevent Flutter?

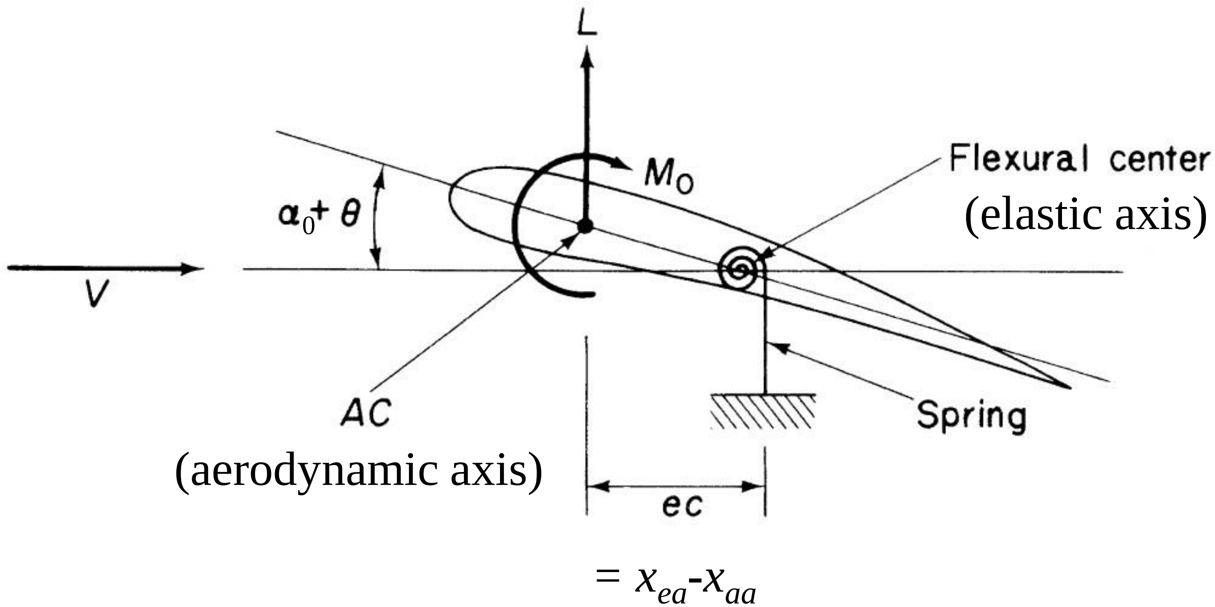
Flutter characteristics of a model are a function of many structural parameters including:

- the shape of the airfoil section,
- the elastic axis position,
- the position of the center of gravity,
- the airfoil section mass and mass moment of inertia about the elastic axis,
- the torsion rigidity,
- the frequency separation between the plunge and rotation modes.

3. Wing divergence

In this case there is only one DOF: rotation.

Divergence occurs when the aerodynamic moment about the elastic axis overcomes the resisting structural moment of the wing and pitch increases until the wing breaks up.



3. Wing divergence
Divergence Equation (2D)

The critical divergence condition is defined as

$$k_\theta \theta = M_{ea} = M_0 + L(x_{ea} - x_{aa})$$

or

$$k_\theta \theta = \frac{1}{2}\rho U^2 S C_{m,0} c + \frac{1}{2}\rho U^2 S \left[C_{l,0} + \frac{\partial C_l}{\partial \alpha}(\alpha_0 + \theta) \right] (x_{ea} - x_{aa})$$

Rearranging and solving for θ, we have

$$\theta = \frac{\frac{1}{2}\rho U^2 S \left[C_{m,0} c + \left(C_{l,0} + \frac{\partial C_l}{\partial \alpha}\alpha_0 \right)(x_{ea} - x_{aa}) \right]}{k_\theta - \frac{1}{2}\rho U^2 S (x_{ea} - x_{aa}) \frac{\partial C_l}{\partial \alpha}}$$

3. Wing divergence

Divergence Equation (2D)

Then, the condition that makes the twist angle θ tend to infinity is

$$k_\theta - \frac{1}{2}\rho U^2 S(x_{ea} - x_{aa})\frac{\partial C_l}{\partial \alpha} = 0$$

The divergence speed becomes

$$U_d = \sqrt{\frac{2k_\theta}{\rho S\left(x_{ea} - x_{aa}\right)\frac{dC_l}{d\alpha}}}$$

3. Wing divergence

How to Increase Divergence Speed?

Increase torsional stiffness:

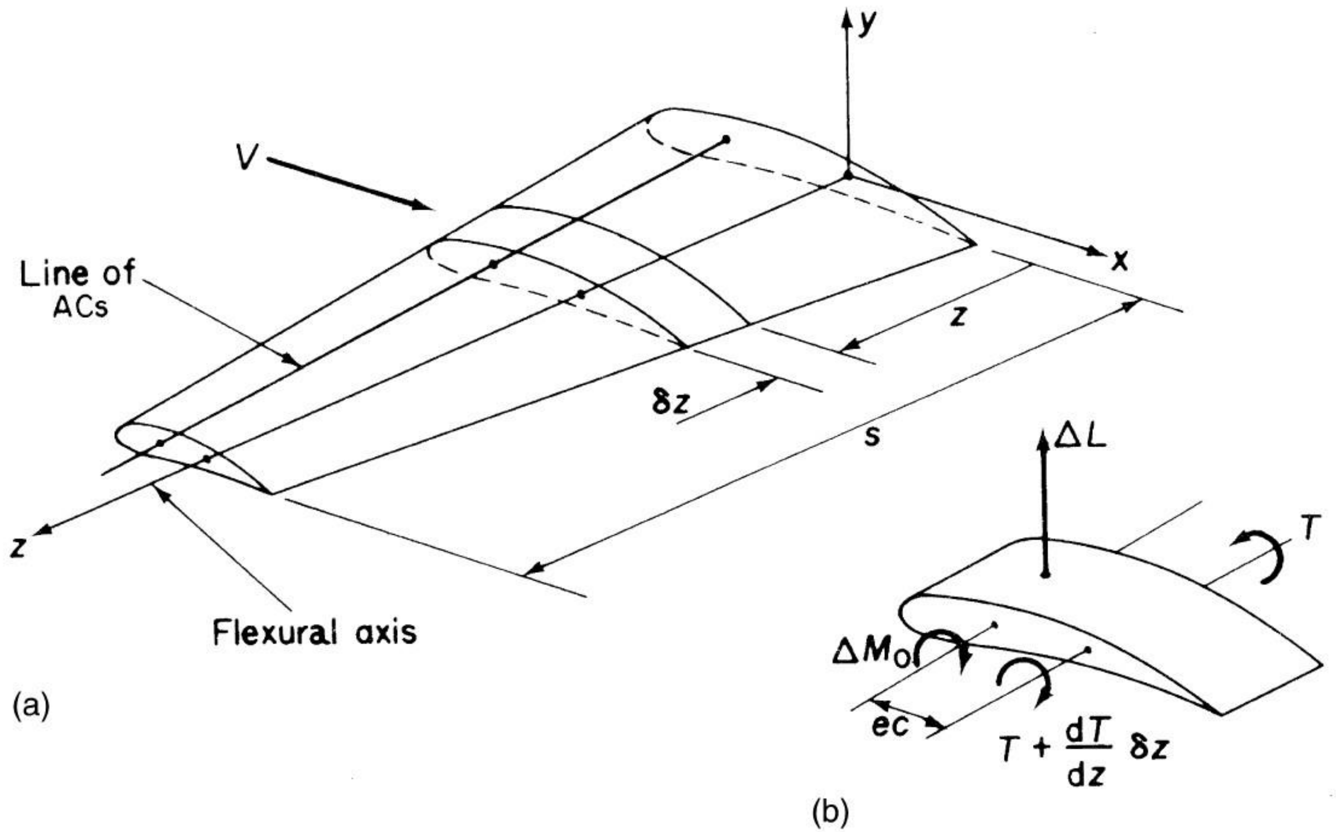
$$k_\theta = (GJ)'$$

where (*GJ*)’ is the torsional rigidity per unit span,

and/or move elastic axis forward (closer to aerodynamic axis).

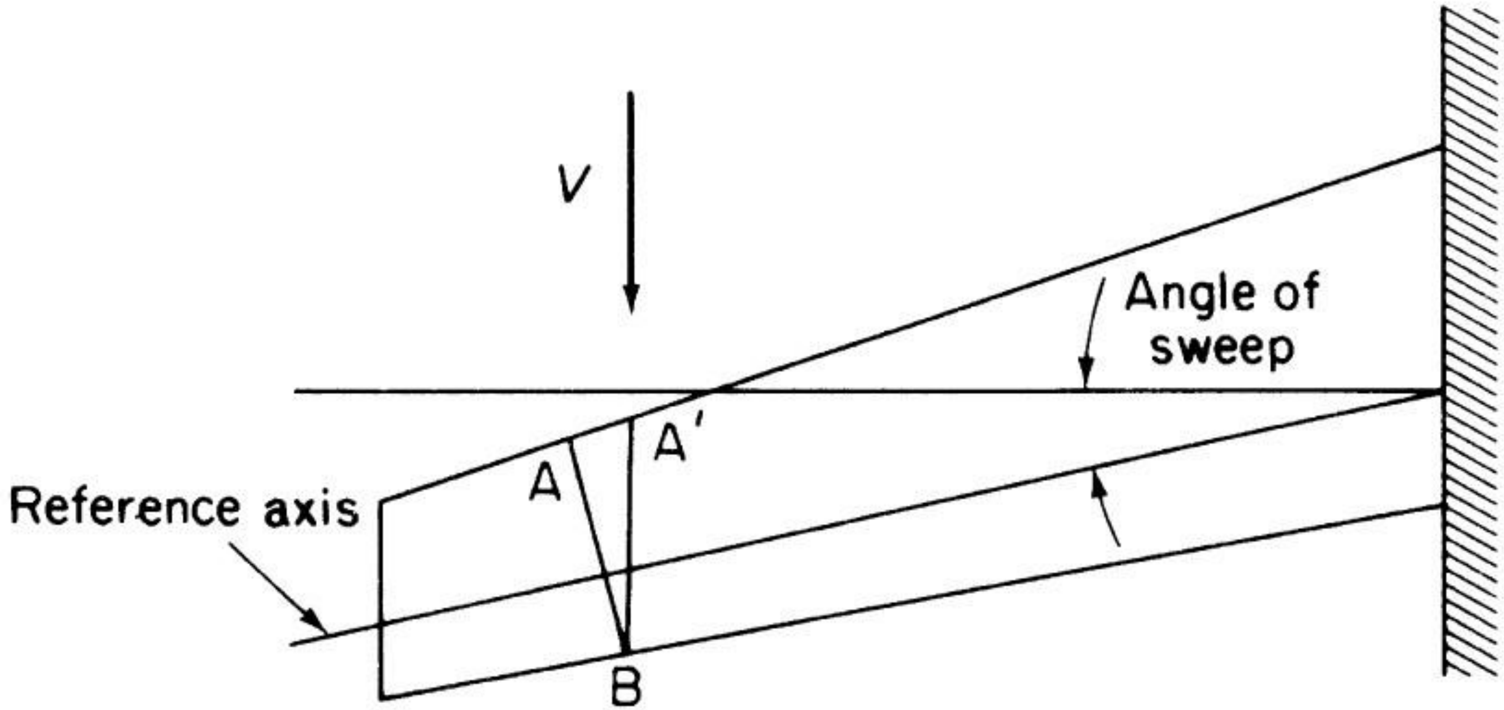
$$U_d = \sqrt{\frac{2k_\theta}{\rho S (x_{ea} - x_{aa}) \frac{dC_l}{d\alpha}}}$$

3. Wing divergence
Finite wing



3. Wing divergence

Swept wing



3. Wing divergence

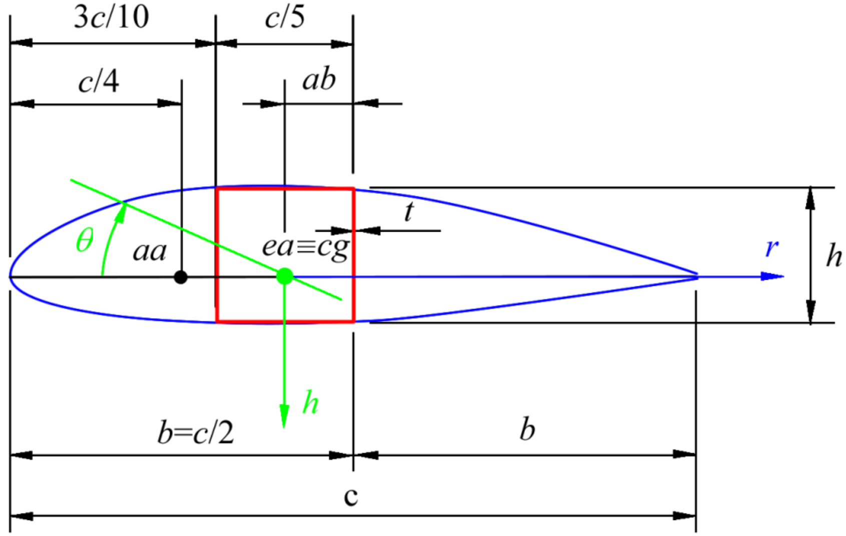
Example 4.01: An aircraft wing has an area of 30 m^2 and an aerodynamic mean chord of 3.0 m. If the aerodynamic centre of the wing is one quarter of the wing chord forward of its flexural centre, its lift-curve slope is 3.5 and the design diving speed of the aircraft at sea level is 200 m/s calculate the minimum required torsional stiffness of the wing. Assume two-dimensional flow and take the air density at sea level to be 1.226 kg/m^3.

The minimum required torsional stiffness will occur when the wing divergence speed is equal to the design

diving speed.

3. Wing divergence

Exemplo 4.02: Pretende-se analisar quanto às características de divergência a secção de asa da figura abaixo. Esta asa é retangular com semi-envergadura $b/2$ = 0,75 m e a sua estrutura é constituída por uma caixa de torção de geometria retangular e paredes finas com altura h = 0,015m, largura 0,2c (onde c = 0,15 m é a corda da asa) e espessura constante t.

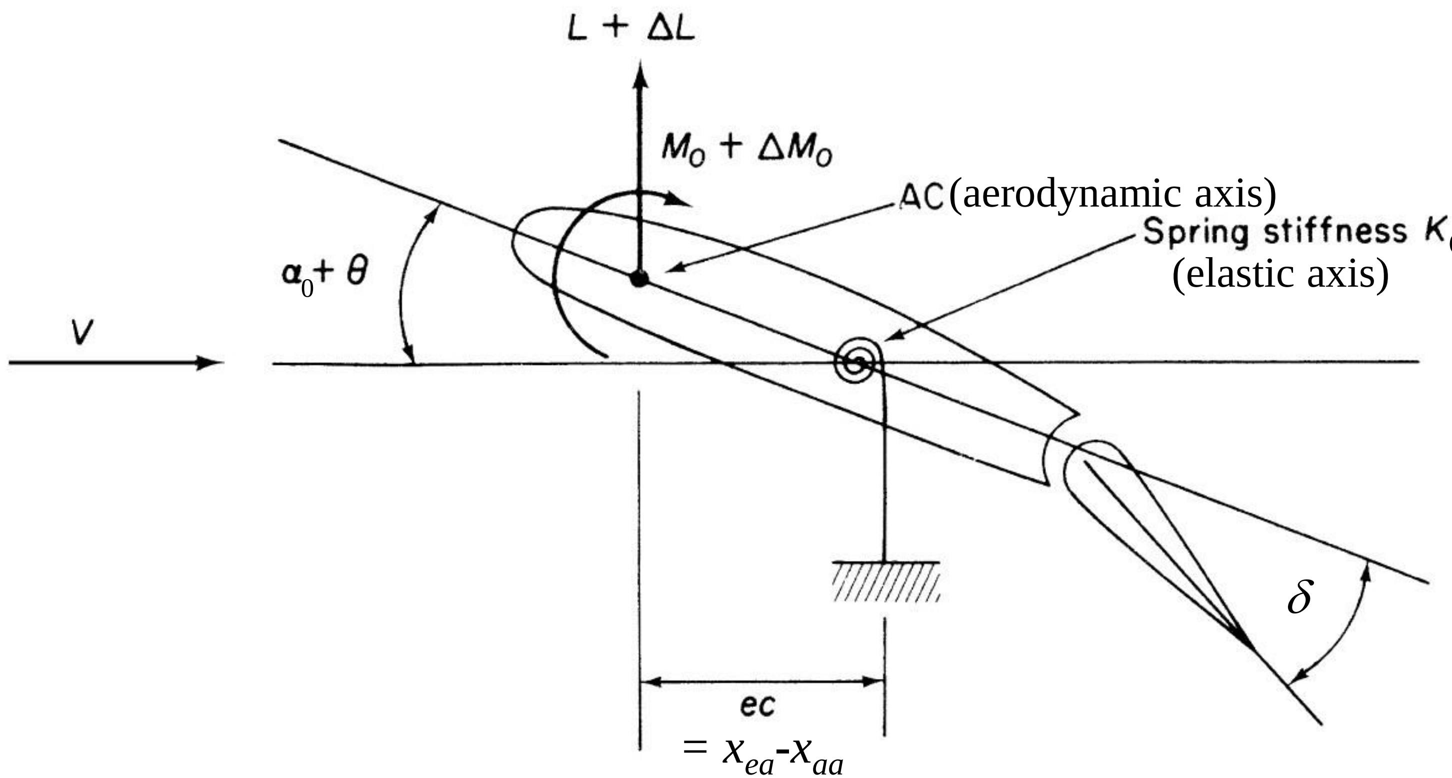


3. Wing divergence

Exemplo 4.02 (continuação): Sabendo que a caixa de torção tem um módulo de corte G = 5 GPa, que o declive de sustentação do perfil alar é $dC_l/d\alpha = 2\pi$ rad^{-1}, que a massa volúmica do ar é ρ = 1,225 kg/m^3 e que ab = 0,1c, determine o valor mínimo da espessura t para que a velocidade crítica de divergência seja igual a 200 m/s.

4. Control Reversal

In this case we have one more DOF, the control surface rotation angle δ, making a 3-DOF problem.



4. Control Reversal

Aileron reversal equation (2D)

When the aileron is deflected by d, there are changes in lift and aerodynamic pitching moment

$$\Delta L = \frac{1}{2}\rho U^2 S\left(\frac{\partial C_l}{\partial \alpha}\theta + \frac{\partial C_l}{\partial \delta}\delta\right)$$

$$\Delta M_0 = \frac{1}{2}\rho U^2 Sc\frac{\partial C_{M,0}}{\partial \delta}\delta$$

The elastic moment must balance the increase in the aerodynamic moment

$$k_\theta \theta = \Delta M_{ea} = \Delta M_0 + \Delta L(x_{ea} - x_{aa})$$

or

$$k_\theta \theta = \frac{1}{2}\rho U^2 S\left[c\frac{\partial C_{M,0}}{\partial \delta}\delta + \left(\frac{\partial C_l}{\partial \alpha}\theta + \frac{\partial C_l}{\partial \delta}\delta\right)(x_{ea} - x_{aa})\right]$$

4. Control Reversal

Aileron reversal equation (2D)

Solving for θ gives

$$\theta = \frac{\frac{1}{2}\rho U^2 S\left[c\frac{\partial C_{M,0}}{\partial \delta} + \frac{\partial C_l}{\partial \delta}(x_{ea} - x_{aa})\right]\delta}{k_\theta - \frac{1}{2}\rho U^2 S(x_{ea} - x_{aa})\frac{\partial C_l}{\partial \alpha}}$$

Substituting this result in the equation for ΔL gives

$$\Delta L = \frac{1}{2}\rho U^2 S\left\{\frac{\partial C_l}{\partial \alpha}\frac{\frac{1}{2}\rho U^2 S\left[c\frac{\partial C_{M,0}}{\partial \delta}+\frac{\partial C_l}{\partial \delta}(x_{ea}-x_{aa})\right]}{k_\theta-\frac{1}{2}\rho U^2 S(x_{ea}-x_{aa})\frac{\partial C_l}{\partial \alpha}} + \frac{\partial C_l}{\partial \delta}\right\}\delta$$

or

$$\Delta L = \frac{1}{2}\rho U^2 S\left\{\frac{\frac{1}{2}\rho U^2 S c\frac{\partial C_{M,0}}{\partial \delta}\frac{\partial C_l}{\partial \alpha}+k_\theta\frac{\partial C_l}{\partial \delta}}{k_\theta-\frac{1}{2}\rho U^2 S(x_{ea}-x_{aa})\frac{\partial C_l}{\partial \alpha}}\right\}\delta$$

4. Control Reversal
Aileron reversal equation (2D)

The lift increment is a linear function of aileron deflection.

Aileron reversal occurs when the increment in lift is zero for a given δ, thus

$$\frac{1}{2}\rho U^2 Sc\frac{\partial C_{M,0}}{\partial \delta}\frac{\partial C_l}{\partial \alpha} + k_\theta\frac{\partial C_l}{\partial \delta} = 0$$

The aileron reversal speed is

$$U_r = \sqrt{\frac{-2k_\theta\dfrac{\partial C_l}{\partial \delta}}{\rho Sc\dfrac{\partial C_{M,0}}{\partial \delta}\dfrac{\partial C_l}{\partial \alpha}}}$$

4. Control Reversal
Aileron effectiveness

Aileron effectiveness is the ratio of the elastic wing lift increment to the rigid wing lift increment

$$Aileron\ effectiveness = \frac{\Delta L}{\Delta L_R}$$

Assim,

$$Aileron\ effectiveness = \frac{\frac{1}{2}\rho U^2 S \left\{ \frac{\frac{1}{2}\rho U^2 Sc \frac{\partial C_{M,0}}{\partial \delta}\frac{\partial C_l}{\partial \alpha} + k_\theta \frac{\partial C_l}{\partial \delta}}{k_\theta - \frac{1}{2}\rho U^2 S(x_{ea} - x_{aa})\frac{\partial C_l}{\partial \alpha}} \right\} \delta}{\frac{1}{2}\rho U^2 S \frac{\partial C_l}{\partial \delta}\delta}$$

or

$$Aileron\ effectiveness = \frac{\frac{1}{2}\rho U^2 Sc \frac{\partial C_{M,0}}{\partial \delta}\frac{\partial C_l}{\partial \alpha} + k_\theta \frac{\partial C_l}{\partial \delta}}{\left[k_\theta - \frac{1}{2}\rho U^2 S(x_{ea} - x_{aa})\frac{\partial C_l}{\partial \alpha}\right]\frac{\partial C_l}{\partial \delta}}$$

4. Control Reversal
Aileron effectiveness

Or in term of the aileron reversal speed and wing divergence speed, we can write

$$Aileron\ effectiveness = \frac{1 - \frac{U^2}{U_r^2}}{1 - \frac{U^2}{U_d^2}}$$

4. Control Reversal

Example 4.03: Calculate the aileron reversal speed corresponding to the minimum required torsional stiffness for the wing of Example 4.01 and determine the aileron effectiveness at a speed of 150 m/s. Take the rate of change of lift coefficient with aileron angle as 0.8 and the rate of change of pitching moment coefficient with aileron angle as –0.25.

From Example 4.01, $K_\theta = 1.93 \times 10^6$ Nm.